Assessment and Selection of Stress Field Models

Continuous Deep Beam Models

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ABSTRACT

Strut-and-tie models (STM) or stress field models (SFM) are commonly used methods for the dimensioning and design of discontinuity zones of reinforced and prestressed concrete structures. These methods are based on the load path of the stress trajectories, and the safety of the region is secured by establishing the equilibrium of the entire model. So, a few equilibrated models can be proposed for a structural member subjected to a load combination but, since no compatibility questions are yet considered, not all provide a good service behaviour.

Based on the finite element method, *Ruiz* and *Muttoni* (2007) presented a new approach that covers that question, helping to evaluate and select stress field models for discontinuity concrete regions. Therefore, it is possible to analyse the response of the structural member at each load increment, helping to establish if the service behaviour of the structure is adequate or not.

Applying non-linear finite element method analysis, this paper intends to evaluate a few continuous deep beam models subjected to a uniform load helping to realize how far from the elastic trajectories stress field models the designers can go when dimensioning deep beams. Some parameters variations are made and the response of the structure, duo to that variation, is documented.

Key Words: Strut-and-tie models, Stress Field models, continuous deep beam, service behaviour, finite element method

1. INTRODUCTION

The designing and detailing of discontinuity zones with Strut-and-Tie Models (STM) or Stress Field Models (SFM) was developed by several authors. Those methods are based on the load path of the stress fields developed in the structural members. *Schlaich* and *Muttoni* have developed some interesting practical guides for the formulation of those methods. They proposed some practical rules on the strut, ties and nodes detailing and provided the minimum energy approach when choosing the most efficient model. However, there have always been some doubts about the service behaviour of those models, especially concerning crack widths.

Several authors have been developing methods of automatic selection of the best model to be adopted in the design of discontinuity zones. Mentioning some of them *Bendsoe* (1995) suggested the ground truss analogy where an n-times statically indeterminate initial strut-and-tie model is generated. The model consists by several nodes connected by bars. Based in the minimum energy criteria, those bars are removed until the most efficient solution is found. *Kostic (2006)* and *Liang (2000)*, based on *Bendoe*´s analogy, proposed different optimization methods on the generation of the most efficient model. *Vitone (*2006) suggested an approach where the behavior off the structure in the different load stages is predicted, helping to choose the most suitable model for the structure. *Lourenço (2010)*, through the nominated *Adaptive Stress Field Models,* suggested a tool for optimization and selection of the best strut and tie model.

However, some of the mentioned methods presents certain limitations. Not all of them give information on the behaviour at the serviceability limit state and not all of them consider the continuity of the concrete. *Muttoni* and *Ruiz* (2007) proposed a different approach based on the finite element analysis that overcomes most of the limitations of the above mentioned analysis.

One of the structural typologies that needs strut-and-tie models or stress field models to be detailed is the continuous deep beam, which will be studied in this paper. Usually, as predicted by many authors, deep beams present some ductility, however it is necessary to evaluate the designer´s freedom in their dimensioning. Applying finite element analysis proposed by *Muttoni* and *Ruiz (2007)*, and a computer program developed by *Miguel Ferreira (2006),* a few continuous deep beams subjected to uniform top load models will be analysed, evaluating, for each load increment, the structure´s response. Redistribution levels of the deep beam will be analysed by variation of some parameters within the different models as the inner level arm and the reinforcement level ate de mid span and mid support.

2. STM SELECTION METHODS

As mentioned before, the STM are based on the load path of the stress trajectories, so there are a few possible STM for the same structure subjected to the same load. As mentioned by *Schlaich* et al (1987), the loads always chose the shortest way leading to shortest deformation.

These general criteria is consensual among several authors and helps choosing the most adequate model for the structure. It can be formulated by:

$$\sum F_i l_i \epsilon_{mi} = Min$$

Where:

- F_i is the force in strut or tie i;
- l_i is the length of the member i;
- ϵ_{mi} is the mean strain of the member i;

As mentioned before several authors developed automatic generation of STM with an adequate service behaviour (*Bendsoe (1996)*, *Vitone (2006)*, *Kostic (2006), Liang (2000) and Lourenço (2010)*).

In this paper, for the analyses of the STM of the continuous deep beam it will be used the *Muttoni* and *Ruiz (2007)*´s *Finite Element (FE) Analyses* concept. Using FE it is possible to overcome some difficulties of some of the previous cited methods as compatibility and the prediction of the behaviour of the structure at serviceability limit state.

3. FINITE ELEMENT METHOD

Muttoni and *Ruiz*´s FE analyses, unlike other more refined ones, requires a limited number of physical parameters. Only the resistances and elasticity modules of the materials are considered, making the analysis simpler.

3.1. CONCRETE MODELLING

It is assumed that the main stress directions are parallel to the main strain directions. Therefore, given the strain field, it is possible to obtain the stress field. The tensile strength of the concrete is neglected and is assumed an elastic- perfectly plastic response duo to compressions (Figure 1).

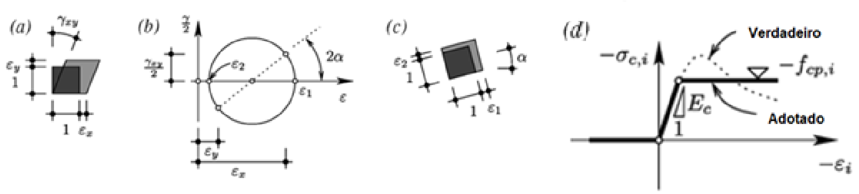

Figure 1 *- (a) concrete element strain; (B) Mohr´s circle and principle strains; (c) directions of principal srains; (d) actual and adopted stress-strain response. (Adapted from Muttoni and Fernández, 2007)*

Concrete´s elasticity module (E_c) is considered independent from its transversal strain as proposed by *Vecchio* et al. (1986). The concrete´s compressive strength is given by:

$$f_{cp} = 3.1\left(f_c^{'}\right)^{\frac{2}{3}}.\eta\left(\varepsilon_j\right)$$

Where $f_c^{'}$ is the cylinder compressive strength. *Muttoni* and *Ruiz* used on this formulation, *Hars (2006)* value to the influence of the transverse strain in concrete strength, $\eta\left(\varepsilon_j\right)$:

$$\eta\left(\varepsilon_j\right) = \frac{1}{0.9 + 30\left(f_c^{'}\right)^{1/3}\varepsilon_j}$$

The implementation of the concrete on the FE model is made by CST elements ("Constant Strain Triangles"). Those elements present constant deformations on its domain, obtained through its displacement field (Figure 2).

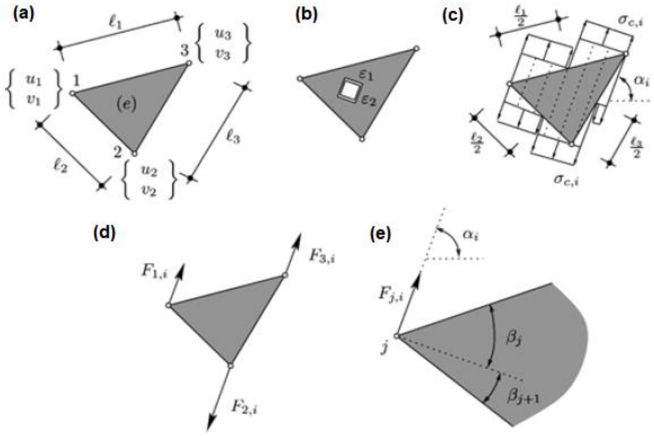

Figure 2 *- (a) displacement field in element; (b) strain field in element; (c) assumed stress field for principal stress i; (d) nodal forces; (e) angle at each node. (Adapted from Muttoni and Ruiz, 2007)*

3.2. STEEL MODELLING

The steel is modelled considering a uniaxial response and the stress-strain graph shows a bilinear behaviour (Figure 3 - c).

In addition to these considerations, when modeling steel, a phenomenon called "Tension Stiffening" is considered. It is a phenomenon defined by the increased stiffness of the shaped steel element due to the surrounding concrete. In the cracked area the tension is all supported by steel, in the area without cracks there is a distribution of the tensions. In order to take this phenomenon into account, the strengths and the areas of the steel and the surrounding concrete were considered.

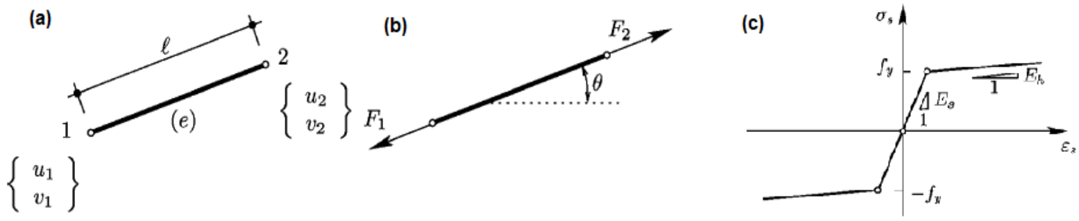

Figure 3 *- (a) displacement field at element; (b) nodal forces and (c) elastoplastic behaviour. (Adapted from Muttoni and Ruiz 2007)*

4. CONTINUOUS DEEP BEAM MODELS

The continuous deep beam studied in this paper has three spans and a height of 12m in order to not limit the level of compressions at the top of the beam. The deep beam has a thickness of 0.25m and is subjected to a uniform load of 1000kN/m. The concrete considered is C40/50 and the steel A500 (Figure 4).

In order to reduce the theoretical influence of the lateral span on the central span the elastic bend moment at the mid support is equated and the dimensions of the spans are find (6m and 7.5m) (Figure 4).

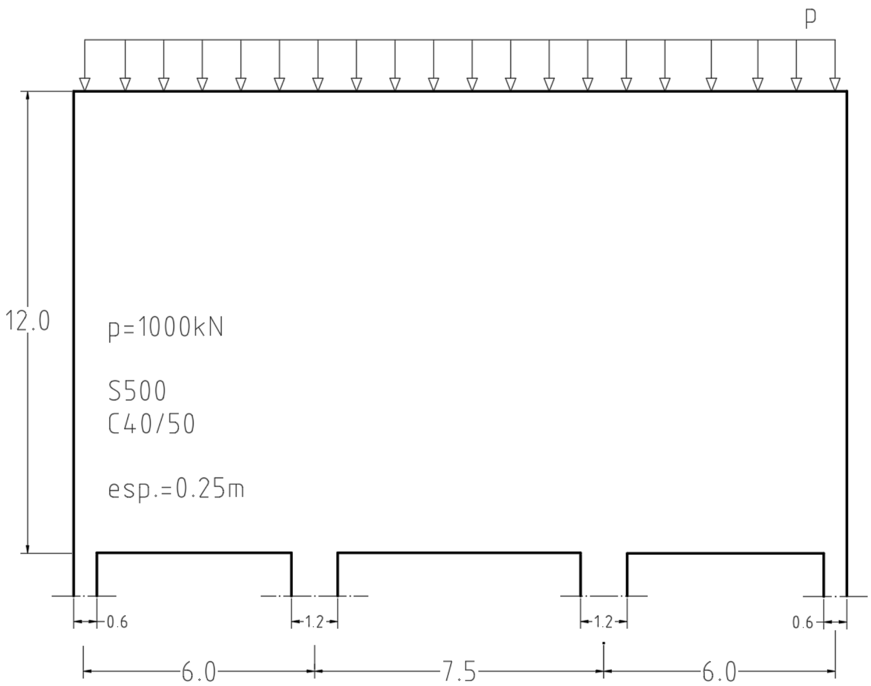

Figure 4 *- Geometry of the continuous deep beam in analysis. (Dimensions in meters)*

To analyse the designer´s freedom two model categories were developed: One where the inner level arm is the same as the elastic stress trajectories (Figure 5) and the variable parameter is the amount of steel reinforcement; and the second where the inner level arm at the mid span is the same as the inner level arm at the mid supports (z=0.7L) and the variable parameter is also the amount of steel reinforcement. For both categories a few models were developed and analysed. To help the evaluation of the models the nomenclature of the models refers to the amount of reinforcement calculated at the mid span of the central span, mid support and lateral span respectively. (Tables 1 and 2).

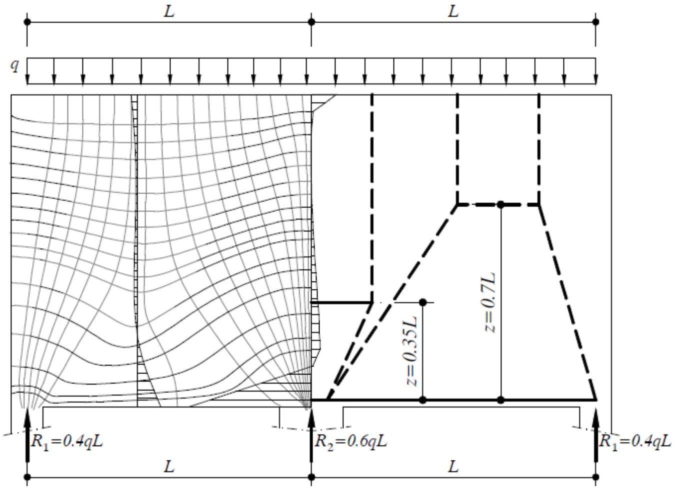

***Figure 5** - Reference strut and tie model based on the elastic trajectories for a continuous deep beam. (Adapted from Lourenço, 2010)*

***Table 1** – Models based on the elastic inner level arm*

(*) – Less than the minimum refinforcement (ρ=0.2%)

External Reaction Value	Central Span	Mid support	Lateral Span	Nomenclature
0	0.0As,reff (*)	7.0As,reff	0.0As,reff (*)	0.0-7.0-0.0
0.2pl	0.0As,reff (*)	6.4As,reff	0.25As,reff	0.0-6.4-0.25
0.35pl	0.5As,reff	2.6As,reff	0.76As,reff	0.5-2.6-0.25
Reference Model	As,reff	As,reff	As,reff	Reference Model
0.45pl	1.4As,reff	0.4As,reff (*)	1.3As,reff	1.4-0.4-1.3
0.5pl	2As,reff	0.0As,reff (*)	1.6As,reff	2.0-0.0-1.6

***Table 2** – Models with the same mid supports and mid span inner level arms (z=0.7L)*

(*) – Less than the minimum refinforcement (ρ=0.2%)

External Reaction Value	Central Span	Mid support	Lateral Span	Nomenclature
0	0.0As,reff (*)	3.8As,reff	0.0As,reff (*)	0.0-3.8-0.0
0.2pl	0.0As,reff (*)	2.2As,reff	0.25As,reff	0.0-2.2-0.25
0.4pl	As,reff	0.6As,reff (*)	As,reff	1.0-0.6-1.0
0.45pl	1.4As,reff	0.2As,reff (*)	1.3As,reff	1.4-0.2-1.3
0.5pl	2.0As,reff	0.0As,reff (*)	1.6As,reff	2.0-0.0-1.6

The evaluations of the models are based on the analyses of the stress level in the reinforcement bars. Since they are directly related to the crack widths it is possible to understand the behaviour of the structure on service loads ($0.5<q/q_d<0.7$).

5. CONTINUOUS DEEP BEAM MODELS ASSESSMENT

5.1. MODELS BASED ON THE ELASTIC INNER LEVEL ARM

The reference model, as expected, has an adequate service behaviour, with relatively low stress levels in the mid support and in the central and lateral mid span steel reinforcement sections. As for the other dimensioned models, it should be noted that a small amount of reinforcement in the middle support generates high levels of tension in the same area. The same happens at the middle of the central and lateral spans. It can be concluded that in these situations the models present a bad behaviour in service, leading to considerable crack widths. Can also be noted that, for the reference model, the STM calculated presents conservative results, as the stresses presented in the following graphics are higher than the results obtained by the FE analysis. (Figures 6 to 9).

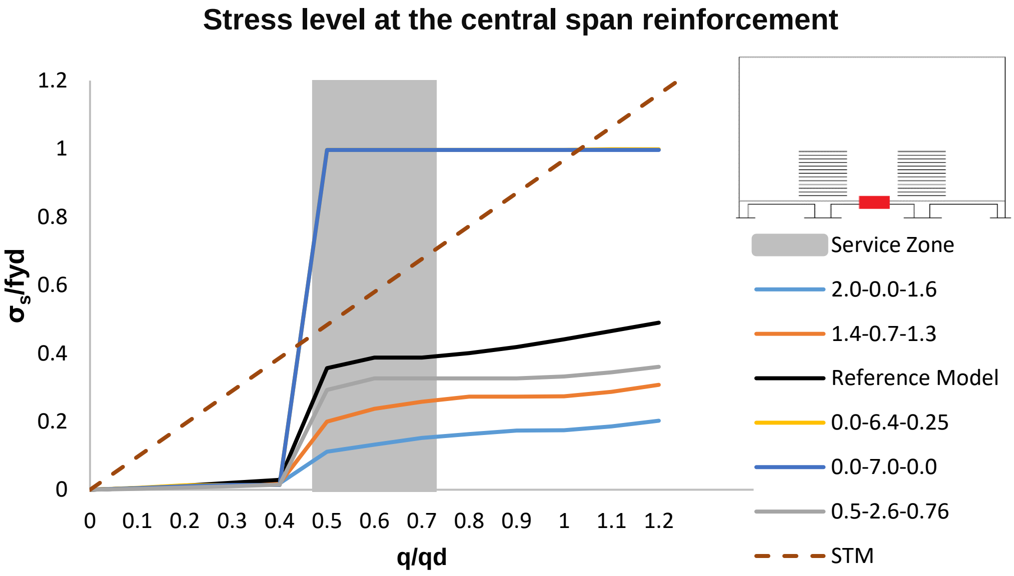

Figure 6 *– Stress level at the central span reinforcement for the models based on the elastic inner arm.*

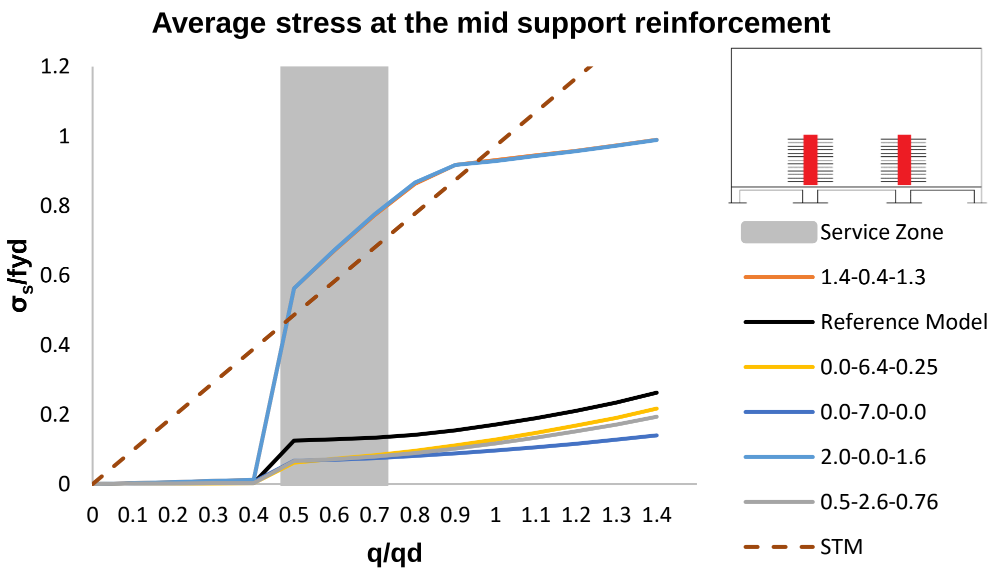

Figure 7 *– Average stress at the mid support reinforcement for the models based on the elastic inner arm.*

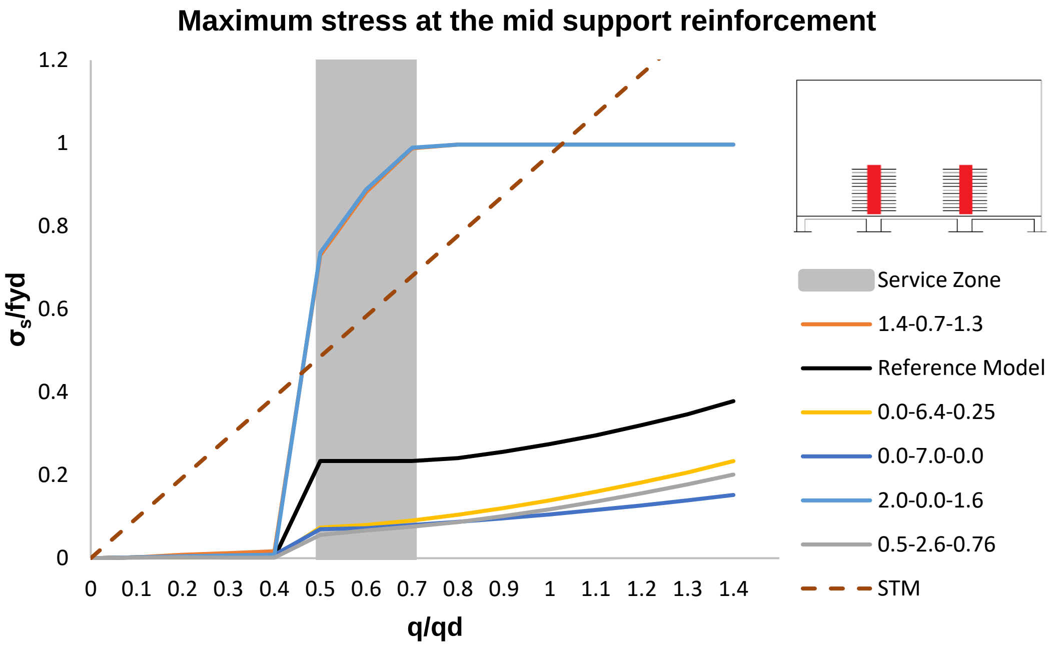

Figure 8 *- Maximum stress at the mid support reinforcement for the models based on the elastic inner arm.*

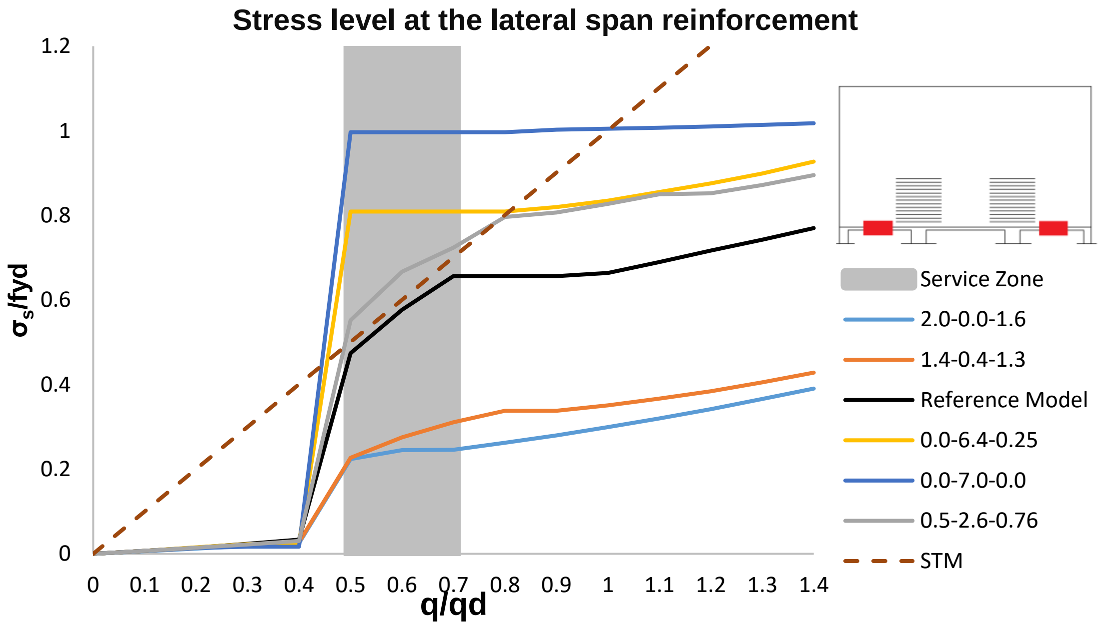

Figure 9 *- Stress level at the lateral span reinforcement for the models based on the elastic inner arm.*

5.2. MODELS BASED ON A UNIQUE INNER ARM (z=0.7L)

The rise of the inner arm in the mid support to 0.7L did not generate adequate results in the service behavior. On one hand, by calculations, the tension on the support is lower than on previous models. On the other, the reinforcement is distributed over a greater height. Those two facts lead to less steel reinforcement amount. As we have already seen in the previous examples, little reinforcement over the mid support generates poor results in service, leading to large crack widths (Figures 10-12).

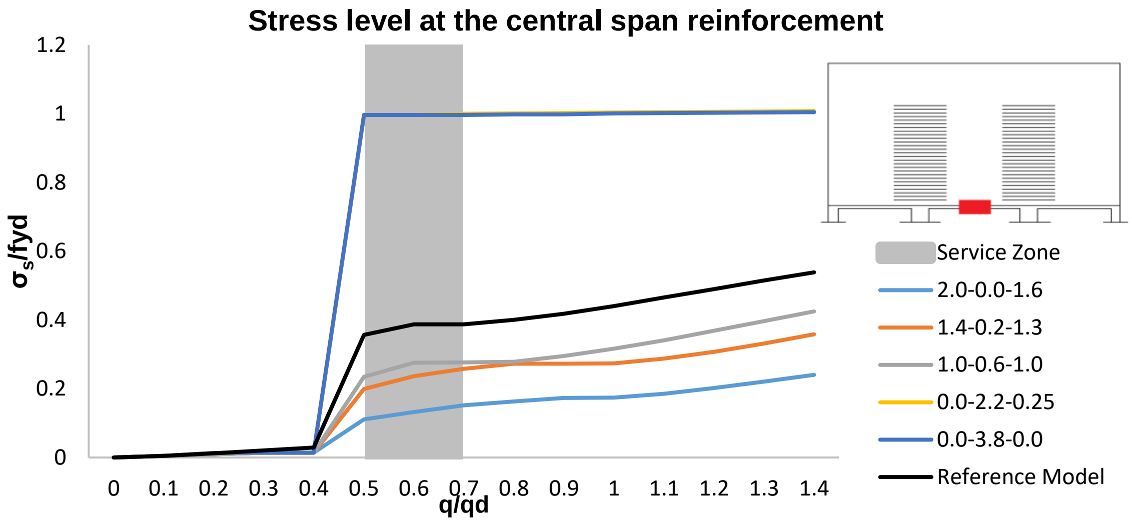

Figure 10 *– Stress level at the central span reinforcement for the models with the same inner arm (z=0.7L).*

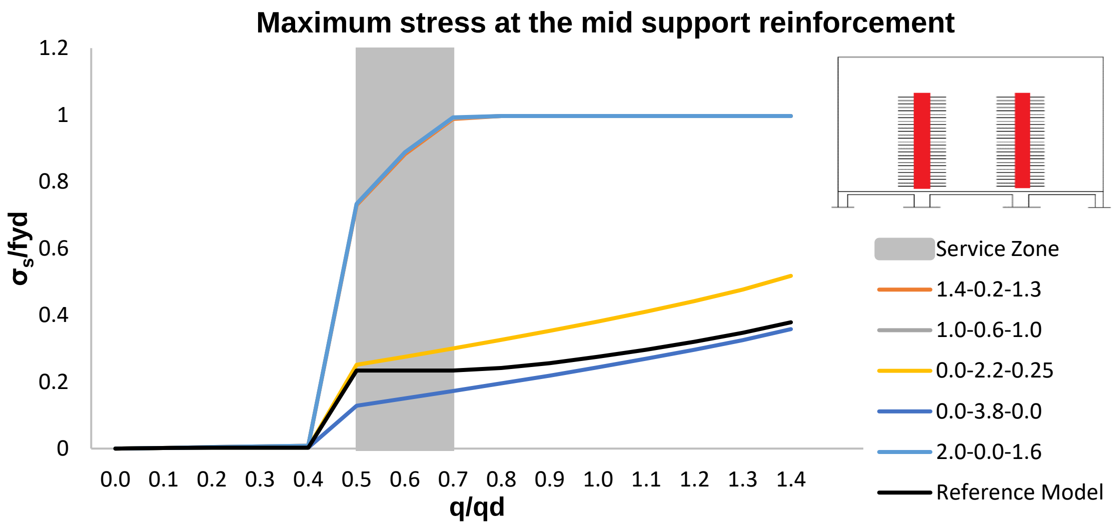

Figure 11 *- Maximum stress ate the mid support reinforcement for the models with the same inner arm (z=0.7L).*

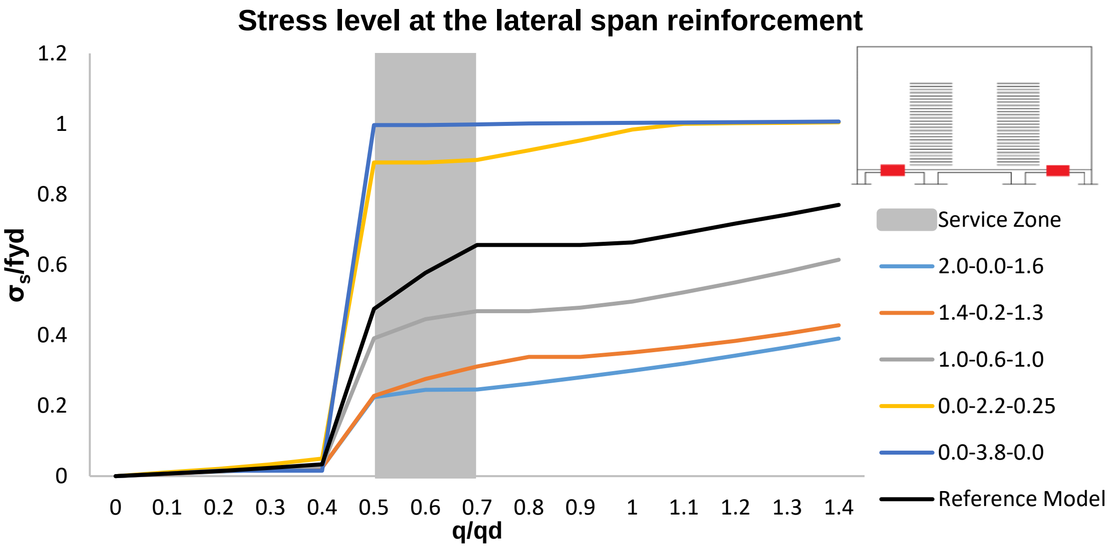

Figure 12 *– Stress level at the lateral span reinforcement for the models with the same inner arm (z=0.7L).*

6. FINAL REMARKS

The analysis of the stresses in the steel reinforcement is a tool for analyzing the service behaviour of the different STM. In continuous deep beams, the results indicate that a small amount of reinforcement in the mid supports generates high stresses in service and possibly large crack widths in this area. The same happens if there is a small amount of reinforcement in the mid span. It can also be concluded that the STM used for dimensioning of the reference model presents conservative results.

The rise of the inner arm above de mid support generated high stress tensions due to the smaller amount of reinforcement needed in that area. Maybe opting to have smaller length distribution of the distributed reinforcement, above the mid support, lead to better results.

Finally, the results demonstrate that big variations from the elastic solution of STM do not generate adequate results in service (Table 3).

***Table 3** - Adequate values for the dimensioning of continuous deep beam STM.*

Parameters	**Models with the elastic trajectories inner arms**	**Models with the same inner arm (z=0.7L)**
External support reaction	0.35pl-0.4pl	-

BIBLIOGRAPHY

ANDERHEGGEN, E – Finite Element Analysis Rigid-Ideal-Plastic Material Behavior, March 1977

BENDSOE, M.P.; SIGMUND, O – Topology Optimization, Theory, Methods and Applications, 2003

BIONDINI, F; BONTEMPI, F; MALERBA, P - Stress path adapting Strut-and-Tie models in cracked and uncracked R.C. elements, Structural Engineering and Mechanics, Vol. 12, No. 6 (2001) 685-698, December 2001.

CAMARA, J. N; COSTA, A; ALMEIDA, J; JÚLIO, E; ALFAIATE, J – Estruturas de Betão – Folhas de Apoio as Aulas, Lisboa

EUROCODE 2 EN 1992-1-1 Design of concrete structures – Part 1.1: General rules and rules for buildings, December 2004

FERREIRA, M.; ALMEIDA, J.; LOURENÇO, M. - Modelling structural concrete with strut-and-tie model combined with 2D finite elements - a model factor for the assessment of strut-and-tie models, fib Symposium 2017, High Tech Concrete: Where Technology and Engineering Meet, Maastricht, June 12-14 2017, pp. 1191-1199

FIP Pratical Design of Structural Concrete, September 1999

KOSTIC, N – Computer Based Development of Stress Fields, 6th International PhD Symposium in Civil Engineering, Zurich, August 23-26, 2006

LIANG, QING QUAN; XIE, YI MIN; STEVEN, GRANT PETRICE - Topology Optimization of Strut-and-Tie Models in Reinforced Concrete Structures Using an Evolutionary Procedure, ACI STRUCTURAL JOURNAL, Title no. 97-S36, March-April 2000

LOURENÇO, M - Adaptive Stress Field Models for Structural Concrete, Tese de doutoramento, Universidade de Lisboa, Lisboa, Fevereiro 2010

LOURENÇO, M; ALMEIDA. J - Adaptive Stress Field Models: Assessment of Design Models, ACI STRUCTURAL JOURNAL, Title no. 110-S09, January-February 2013

LOURENÇO, M; ALMEIDA, J - Adaptive Stress Field Models: Formulation and Validation, ACI STRUCTURAL JOURNAL, Title no. 110-S08, January-February 2013

VITONE, A; PALMISANO, F;VITONE, C - Load Path Method (LPM) in Detailing Design, Proceedings of the 2nd International Congress, Session 3 – Modelling and design, Naples, Italy, June 5-8, 2006.

MARTI, P – Theory of Structures, Fundamentals Framed Stuctures Plates and Shells, Zürich, February 2013.

MUTTONI, A; SCHWARTZ, J; THÜRLIMANN, B – Design of Concrete Structures with Stress Fields, Basel; Boston; Berlin: Birkhauser, 1997.

RUIZ, MIGUEL FERNÁNDEZ; MUTTONI, A - On Development of Suitable Stress Fields for Structural Concrete, ACI STRUCTURAL JOURNAL, Title no. 104-S48, July-August 2007

SCHLAICH, J; SCHÄFER, K – Design and Detailing of structural concrete using strut-and-tie models, The Structural Engineer, v69, No.6, March 1991

SCHLAICH, J; SCHÄFER, K; JENNEWEIN, M - Toward a Consistent Design of Concrete, PCI JOURNAL, May-June, 1987

VECCHIO, F. J; COLLINS, M. P. - The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected do Shear, ACI JOURNAL, Title no. 83-22, March-April 1986